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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number TWO (Due: Sat. at 1pm October 13)

Ayman Badawi

QUESTION 1. (i) Let $G =< a >$ be an infinite cyclic group where $a \in G$. Prove that a and a^{-1} are the only generators of G.

(ii) Let G be a finite cyclic group of order 44×99. How many generators does G have? How many proper subgroups does G have?

(iii) Let F be a finite cyclic group with more than one element and L be an infinite cyclic group. Prove that $K = F \oplus L$ is never a cyclic group

(iv) Let F, L be infinite cyclic groups. Prove that $H = F \oplus L$ is never a cyclic group.

(v) Let D be a finite cyclic group of order m and H be a finite cyclic group of order n. Prove that $L = D \oplus H$ is cyclic if f $gcd(m, n) = 1$

(vi) Let a, b be elements of a group G such that $|a| < \infty$ and $|b| < \infty$. Prove that $|aba^{-1}| = |b|$ and $|ab| = |ba|$

(vii) Find an example of an abelian group, say G, such that G has two elements a, b with $|a| = \infty$ and $|b| = \infty$ but $|ab|$ is finite and $|ab| > 1$. Now let a, b be elements of an abelian group G such that $|a| < \infty$ and $|b| = \infty$. Prove that $|ab| = \infty$.

(viii) Let $D = \{4, 8, 12, 16, 20, 24\}$. Prove that D under multiplication module 28 is a group. Find e, and find the inverse of each element of D. Is D cyclic? If yes, find a generator of D.

Faculty information

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